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VIBRATION CONTROL OF MACHINES BY USE OF SEMI-ACTIVE DRY FRICTION DAMPING

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A semi-active device for the vibration control of machinery foundations or for ride control of vehicles is proposed. A semi-active damper utilising dry friction is employed, with balance logic, a class of sequential damping, being used to minimise the force transmitted. The friction force applied to the mass is controlled so as to cancel the spring force, which is only possible when these forces act in opposite directions. Otherwise, the friction force must be set to zero, which is quite achievable, unlike the case of a viscous damper. The friction damper can be controlled so as to mimic the addition of a viscous damper. Both the ideal case of instantaneous switching and a more realistic model with finite switching time are studied. The results of numerical simulations show that a successful implementation of this strategy could produce a significant reduction of the forces transmitted to the foundation by a machine subject to a rotating imbalance force or of the mean square acceleration in a car riding on a randomly profiled road.

1. INTRODUCTION

Isolation of passenger and cargo from terrain induced shock and vibration is the important task of the suspension of any ground vehicle. Also the reduction of machinery transmitted loads to the supporting structure is usually obtained by suspension units consisting of springs and dampers. Most suspensions units are passive units (not requiring any external power). The vibration isolation performance of most passive suspensions, both linear and non-linear, is rather limited. Their transmissibility factors show that low damping gives good isolation at high frequency but poor resonance characteristics, whilst higher damping results in good resonance isolation at the expense of high frequency performance.

By using hydraulic or pneumatic power supply and servoactuators controlled by feedback signals, it is possible to produce active suspensions. In this case the optimum transmissibility has no resonance amplification and the suspension performance is superior to any passive system throughout the frequency range [1–4]. But they are more complex, expensive and less reliable than the passive suspensions.

A compromise between passive and active types are "semi-active" suspension systems [3–9]. They have an active damper in parallel with a passive spring. The damping characteristics are controlled merely by modulation of fluid-flow orifices or of friction forces, based on a scheme involving feedback variables. For practical reasons it is

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important that the feedback signals be relative displacement and relative velocity across the suspension, since these quantities can be measured directly even for a moving vehicle.

In the case of semi-active suspensions with sequential damping, as introduced by Federspiel [10] the damper force is zero or rather very small as long as the sprung mass is moving away from its static equilibrium position, suddenly increases when the stroke changes from bound to rebound and then gradually decreases when the system is returning to the static equilibrium position. The basic idea of this strategy is to balance the elastic force by the damping force in order to reduce or even to cancel the forces transmitted through the suspension as long as the spring force and damper force act in opposite direction.

Some mathematical models for sequential damping have been proposed and analyzed [11–14] showing better vibration isolation properties than the passive damping, both for deterministic and random excitations.

In this paper a more general mathematical model for sequential damping is presented and in particular the free and forced vibrations for the case of sequential dry friction [14, 15]. In practice the balancing of a linear spring force can be achieved in a semi-active system by means of a continuously variable viscous damping [7] or by means of a spring loaded friction brake, the proposal of the authors. This is possible only when the relative displacement and velocity are of opposite sign. Theoretically one can consider an instantaneous switching of the friction force form zero to the demanded value but a more realistic model should take into account a finite switching time [16]. A controllable friction damper with finite switching time has been also reported to the produce a satisfactory reduction of the response displacement in a semi-active seismic isolation system [17].

2. SEQUENTIALLY DAMPED OSCILLATORS

The aim of the sequential damping logic is to oppose the spring force and thus reduce the acceleration of the body compared to the passive case. One strategy is to attempt to achieve zero acceleration. It is only possible to balance the spring and damper forces when relative displacement and relative velocity have opposite sign.

The sequential damping force will therefore be considered as a function of the relative displacement x and relative velocity $\dot{x}$, having the general form:

$$D(x, \dot{x}) = c\dot{x} + [1 - \operatorname{sgn} x\dot{x}]d_1(x)d_2(\dot{x})/2, \qquad c \geqslant 0, \tag{2.1}$$

where d_1 and d_2 are functions of displacement and velocity respectively. $d_1(x) \geqslant 0$, $\dot{x}d_2(\dot{x}) \geqslant 0$ and $d_1(x) = d_2(\dot{x}) = 0$ if and only if $x = \dot{x} = 0$. Moreover, $d_1(x)$ and $d_2(\dot{x})$ are real continuous functions (except possibly at $\dot{x} = 0$) and monotonous on each x and $\dot{x}$ semi-axis.

The motion of a sequentially damped SDOF oscillator having sprung mass M, linear spring force Kx, and absolute displacement x_1 is given by

$$M\ddot{x}_1 + D(x, \dot{x}) + Kx = P(t), \tag{2.2}$$

where $P(t)$ is the exciting force and $x_1 = x + x_0$, x_0 being the input displacement. When the sprung mass is a machine acted on by an imbalance force $P(t)$, the transmitted force to the foundation is

$$F(t) = -[D(x, \dot{x}) + Kx]. \tag{2.3}$$

For an imposed displacement $x_0(t)$ of the unsprung mass, as in a car suspension, the absolute acceleration of the sprung mass is

$$\ddot{x}_1 = -(1/M)[D(x, \dot{x}) + Kx], \tag{2.4}$$

where x is the relative displacement $x_0 - x_1$.

In both cases, the vibration isolation problem implies the reduction of the elastic and damping force resultant across the suspension. In a semi-active design, this can be achieved by controlling the damping force so as to balance the elastic force when these forces act in opposite direction, i.e., when $x\dot{x} < 0$, and by setting the damping force to a minimum value (possibly zero) when $x\dot{x} > 0$. From (2.1) and (2.3) it is obvious that a perfect semi-active balance is obtained if

$$c\dot{x} + d_1(x)d_2(\dot{x}) = -Kx \qquad \text{for} \qquad x\dot{x} < 0. \tag{2.5}$$

The simplest physically realizable solution of the functional equation (2.5) is obtained for

$$c = 0, \qquad d_2(\dot{x}) = \operatorname{sgn}(\dot{x}), \qquad d_1(x) = K|x|. \tag{2.6}$$

In order to reduce body acceleration, energy dissipation should take place only when the friction force opposes the spring force i.e., when $x\dot{x} < 0$. This can be achieved by a controllable friction damper. For a friction coefficient μ, the normal force F_p, applied on the friction plates, must be controlled by a servoactuator such that

$$F_p = \begin{cases} 2\alpha(K/\mu)|x| & \text{ON i.e., when} \quad x\dot{x} < 0, \\ 0 & \text{OFF i.e., when} \quad x\dot{x} > 0, \end{cases} \tag{2.7}$$

where 2α is the gain. The reason for this notation for the gain becomes apparent when equation 2.12 is derived.

The damping characteristic of the system is "on-off" and is given by

$$F_D(x, \dot{x}) = \begin{cases} 2\alpha K|x| \operatorname{sgn} \dot{x} & \text{for} \quad x\dot{x} < 0, \\ 0 & \text{for} \quad x\dot{x} > 0. \end{cases} \tag{2.8}$$

Equations (2.4) and (2.8) show that in the ON condition complete balance is achieved with $\alpha = 0{\cdot}5$. In order to avoid damper 'lock up' (no motion across the damper) which could occur when $\dot{x} = 0$, $x \neq 0$ and $\ddot{x} = 0$, it is sufficient to ensure that friction force produced in this condition is less than the spring force:

$$2\alpha D|x| < K|x|, \tag{2.9}$$

or $\alpha < 0{\cdot}5$. In simulations complete balance ($\alpha = 0{\cdot}5$) was considered and lockup ignored with the understanding that in an experimental case α would have to be slightly less than $0{\cdot}5$.

Equation (2.8) implies an instantaneous switching between the "on" and "off" settings of the friction damper. A more realistic model must take into account a finite switching time [8], [16, 17]. α is now replaced by a demanded value α_{dem} since the force achieved is not in general that demanded by the logic. The variation of the damping force F_d is described by the first order equations:

$$T_c\dot{F}_d + F_d = 2\alpha_{dem}Kx \qquad \text{if} \qquad x\dot{x} < 0, \tag{2.10}$$

$$T_c\dot{F}_d + F_d = 0 \qquad \text{if} \qquad x\dot{x} < 0, \tag{2.11}$$

where T_c is the time constant for the switch.

The equation of motion (2.2) with the addition of a low level of viscous damping to the damping term of equation (2.12) can when $x\dot{x} < 0$ be written in the form:

$$M\ddot{x}_1 + c_{min}\dot{x} + \alpha_{dem}K|x| \operatorname{sgn} \dot{x} + (1 - \alpha_{dem})Kx = P(t), \tag{2.12}$$

where F_D in equation (2.12) has been split into two terms, one a dissipative term and the other a (negative) spring term. Equation (2.12) can be regarded as describing the motion of an oscillator having the dissipative characteristic $D_1(x, \dot{x}) = c_{min}\dot{x} + \alpha_{dem}K|x| \operatorname{sgn} \dot{x}$ and the elastic characteristic $E(x) = (1 - \alpha_{dem})Kx$. This form shows the "weakening" of the spring term caused by partial balance, since the spring term is reduced in magnitude by a factor $1 - \alpha_{dem}$. This reduces the natural frequencies of the system.

By using the following notation $\omega = \sqrt{KM}$, $\zeta_{min} = c_{min}2\sqrt{KM}$, $\tau = \omega t$ and introducing a as a unit of length,

$$y(\tau) = x(\tau/\omega)/a, \qquad y'(\tau) = \mathrm{d}y/\mathrm{d}\tau, \qquad z(\tau) = P(\tau/\omega)/aM\omega, \tag{2.13}$$

the equation (2.12) can be written in the following dimensionless form:

$$y'' + \delta_1(y, y') + (1 - \alpha_{dem})y = z(\tau) - \ddot{z}_{in}, \tag{2.14}$$

where the dimensionless absolute deflection is $y + z_{in}$ and

$$\delta_1(y, y') = 2\zeta_{min}y' + \alpha_{dem}|y| \operatorname{sgn} y', \qquad \alpha_{dem} < 1 \tag{2.15}$$

is the dissipative characteristic of the system.

In what follows, for convenience the dimensionless parameters will be referred to by the corresponding physical parameters (e.g., τ as time, y as relative displacement, $\delta(y, y')$ as damping force, $\psi(y) = (1 - \alpha_{dem})y$ as elastic force). In order to study the vibration isolation performances of semi-active dampers with dry friction, both single-degree-of-freedom and two-degrees-of-freedom systems will be considered. These model machinery foundations and car suspensions respectively.

3. RESPONSE WITH SEMI-ACTIVE DRY FRICTION AND IMPOSED HARMONIC MOTION

The behaviour of the damping force as the relative displacement y is varied throughout an harmonic cycle is important from the practical point of view since most testing machines provide such a relative motion between the mounting ends of the shock absorber. For an imposed cycle motion

$$y(\tau) = Y_0 \sin \nu\tau, \tag{3.1}$$

the total damping force

$$\delta(y, y') = 2\zeta_{min}y' + [1 - \operatorname{sgn}(yy')]\alpha_{dem}|y| \tag{3.2}$$

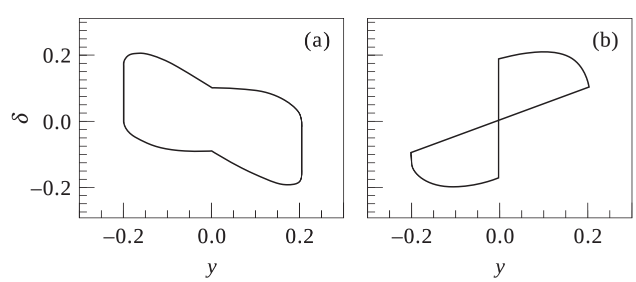

Figure 1. Damping force versus (a) displacement (b) velocity.

varies as shown in Figures 1a and 1b in terms of the relative displacement $y(\tau)$ and relative velocity $y'(\tau)$, respectively. ζ_{min} and α_{dem} were taken as 0·25 and 0·45 respectively. The energy loss per cycle is

$$\Delta w = \int_0^{2\pi - \nu} \delta(y, y')y' \,\mathrm{d}\tau = 2Y_0^2(\pi\nu\zeta_{min} + \alpha_{dem}). \tag{3.3}$$

Form equation (3.3) one can see that the dissipative effect of the additional semi-active dry friction could be very important for low frequency imposed harmonic motions and becomes less important when the frequency of the imposed motion increases. The relative damping coefficient ζ_{eq} of a linear system which provides the same energy loss per cycle as the semi-active damper is given by

$$\zeta_{eq} = \zeta_{min}(1 + \alpha_{dem}/\pi\nu\delta_{min}). \tag{3.4}$$

For the "on-off" damping characteristic $\zeta_{eq} < 1/\pi\nu$ since $\alpha_{dem} < 1$. The maximum absolute value (peak value) of the damping force is reached in a time interval

$$\Delta\tau = (1/\nu)[\pi/2 - \operatorname{arctg} \alpha_{dem}/\nu\zeta_{min}], \tag{3.5}$$

from the instantaneous switch of the dry friction force from zero to its demanded value and is given by

$$\hat{\delta} = |\delta(y, y')|_{max} = 2\zeta_{min} Y_0 \nu\sqrt{1 + \alpha_{dem}^2/(\nu\zeta_{min})^2}. \tag{3.6}$$

Since $\alpha_{dem} < 1$ the peak value of an "on-off" semi-active dry friction damper force will be less than $2Y_0$. For the same energy loss per cycle, the peak value of the "on-off" semi-active damper force should be π times greater than the peak value of the linear equivalent damping force, independent of the cycle frequency.

4. FREE VIBRATIONS

The phase trajectories of the system (2.14) can be determined analytically for the "on-off" damper. In this case, the free vibrations of the system are described by the Cauchy problem

$$\left\{\begin{array}{ll} y_1' = y_2, & y_2' = -\alpha_{dem}|y_1| \operatorname{sgn} y_2 - (1 - \alpha_{dem})y_1, \\ y_1(0) = y_1^0, & y_2(0) = y_2^0. \end{array}\right\} \tag{4.1}$$

The analytical expressions of the phase trajectories are

$$\left\{\begin{array}{llll} y_1^2 + y_2^2 = y_1^{02} + y_2^{02} & \text{if} & y_1^0 y_2^0 \geqslant 0, & y_2^0 \neq 0, \\ (1 - \alpha_{dem})y_1^2 + y_2^2 = (1 - \alpha_{dem})y_1^{02} + y_2^{02} & \text{if} & y_1^0 y_2^0 \geqslant 0, & y_1^0 \neq 0. \end{array}\right\} \tag{4.2}$$

These trajectories are piecewise circular and elliptic curves as shown in Figure 2 for various initial conditions. The ratio between two successive maxima or minima of the free motion relative displacement $y(\tau)$ is constant and is given by $\rho = 1/(1 - \alpha_{dem})$. Therefore, in this case one can define a decrement of the free motion which characterizes the system damping in the same way as the logarithmic decrement does for linear systems.

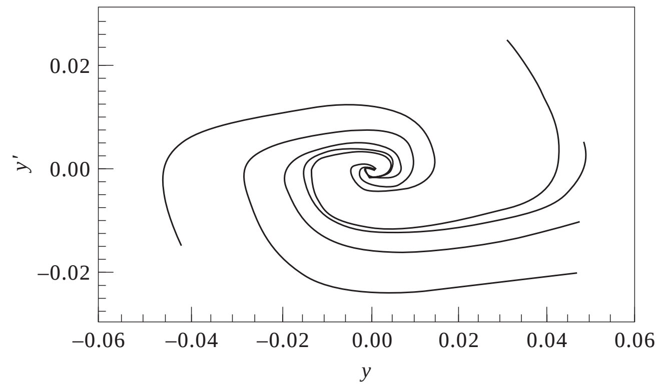

Figure 2. Free oscillations, various initial conditions.

5. SHOCK ABSORBING PROPERTIES

In this section the shock absorbing properties of a single-degree-of-freedom vibration isolation system with sequential dry friction will be compared to those of an optimal linear system. It can be shown that the optimum value of the relative damping coefficient ζ which minimizes the peak value $\hat{y}_1''$ of the transmitted force in the case of a single-degree-of-freedom linear isolation system excited by a Dirac impulse $\delta(\tau)$ is $\zeta_0 = 0{\cdot}25$ [12]. Since the first sequence of the motion described by equation (2.14) for $z(\tau) = \delta(\tau)$ and initial conditions $y(0) = y'(0) = 0$ is always governed by the linear equation, one can obtain the same optimum value for $\hat{y}_{1opt}''$ taking $\zeta_{min} = \zeta_0$ in the sequential damping characteristic (2.15). The effect of the additional sequential damping is then observed by comparing the evolution of the motion after the first peak was reached.

In Figures 3a and 3b there are plotted the time histories of the free vibrations of the passive (optimal linear) system with $\zeta = 0{\cdot}25$ and of the semi-actively controlled system with the same viscous linear damping and semi-active dry friction ($\zeta_{min} = 0{\cdot}25$, $\alpha_{dem} = 0{\cdot}45$). As shown in Figure 3b, the free motion of the system with semi-active dry friction looks like that of an almost critically damped linear system. Nevertheless, the r.m.s. value of the transmitted force from the sprung mass to the system base is significantly lower than in the case of a critically damped linear system. For example, in the case of the passive system with $\zeta = 0{\cdot}7$, the peak value of the transmitted force is $\hat{y}_1'' = 1{\cdot}4$, i.e., 1·7 times greater than the optimum value $\hat{y}_{1opt}'' = 0{\cdot}82$ [12].

6. HARMONIC EXCITATION

6.1. TIME HISTORY

The steady state solution of the equation (2.14) with harmonic excitation $z(t) = Z \sin \nu\tau$ has been determined by numerical integration using Newmark's method [18]. In Figures 4a and 4b there are plotted the time histories of the transmitted force $y_1''(\tau)$ and of the damping force $\delta(y, y')$ in the case of the passive system with $\zeta = 0{\cdot}25$ and of the semi-active system with $\zeta_{min} = 0{\cdot}25$, $\alpha_{dem} = 0{\cdot}45$, for $Z = 0{\cdot}2$ and $\nu = 1$ (i.e., when the excitation frequency is equal to the undamped natural frequency of the system). As one can see, the additional semi-active dry friction leads to an important reduction of the transmitted force (46% for the peak and 54% for the r.m.s. value) for only 3% increase of the damping force peak value.

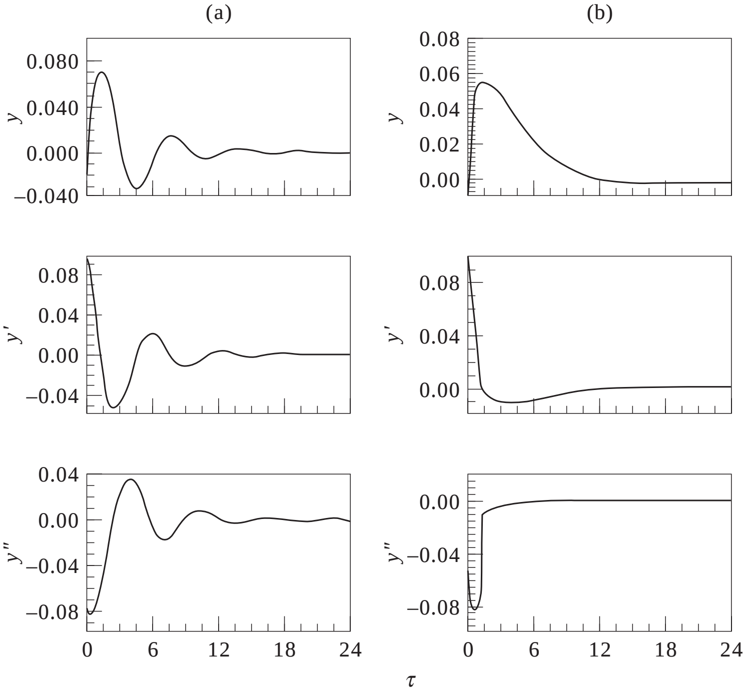

Figure 3. Free oscillations versus time with $\zeta = 0{\cdot}25$, $\alpha_{dem} = 0{\cdot}45$ for (a) passive and (b) semi-active cases.

6.2. FREQUENCY RESPONSE FUNCTIONS

Since one is interested in the reduction of the loads transmitted from the spring mass to the system base, the force transmissibility factor will be compared for the passive and semi-active systems considered above. For the passive system the acceleration ratio $T(v)$ is given by

$$T(v) = \hat{y}_1'' Z = \sqrt{2}\tilde{y}_1/Z, \tag{6.1}$$

where $\hat{y}_1''$ and $\tilde{y}_1''$ are the peak and r.m.s. values of $y_1''(\tau)$, respectively.

In the case of a semi-active system one can define similar frequency response functions which, in general, depend on both the amplitude and frequency of the input:

$$\hat{T}(v, Z) = \hat{y}_1''/Z, \qquad \tilde{T}(v, Z) = \sqrt{2}\tilde{y}_1''/Z. \tag{6.2}$$

Since the equation of motion (2.14) of the system with semi-active dry friction damping is piecewise linear, the frequency response functions (6.2) are independent of the input amplitude Z, but they are not equal as in case of a passive system.

In Figure 5 are plotted the frequency response function $T(v)$ for the passive system with $\zeta = 0{\cdot}25$ and for the semi-active system with $\zeta_{min} = 0{\cdot}25$, $\alpha_{dem} = 0{\cdot}45$. From these diagrams an important resonance shifting toward the lower frequency range results when semi-active dry friction damping is added. This effect is very convenient when a low frequency tuned vibration isolation system is sought, as usual in machinery foundations. Another important feature of semi-active damping is the remarkable reduction of the r.m.s. force transmissibility factor in the resonance range of the initial passive system. For $v > 2$ and same value of ζ the transmissibility factor of both linear and non-linear systems are practically equal. This aspect is very important since for low values of ζ passive suspensions are very efficient vibration isolators in the higher frequency range.

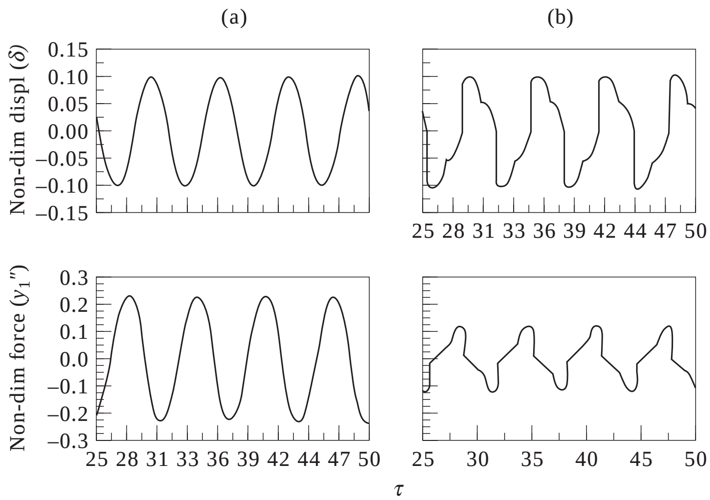

Figure 4. Harmonic excitation, $v = 1$. Damping forces (δ) and transmitted force (y_1'') versus time with $\zeta = 0{\cdot}25$, $\alpha_{dem} = 0{\cdot}45$ for (a) passive and (b) semi-active cases.

7. VEHICLE SUSPENSION WITH SEMI-ACTIVE DRY FRICTION

7.1. MATHEMATICAL MODEL

For the quarter-car model with viscous damping and semi-active dry friction, as shown in Figure 6, the equations of motion are:

$$M_2\ddot{x}_2 + D(x, \dot{x}) + K_2(x) = 0,$$

$$M_1\ddot{x}_1 - D(x, \dot{x}) - K_2(x) + K_1(x_1 - r) = 0, \tag{7.1}$$

where

$$D(x, \dot{x}) = \left\{\begin{array}{lll} c_{min}(\dot{x}) + 2\alpha_{dem}K_2|x| \operatorname{sgn}(\dot{x}) & \text{if} & x\dot{x} < 0, \\ c_{min}\dot{x} & \text{if} & x\dot{x} > 0, \end{array}\right\} \tag{7.2}$$

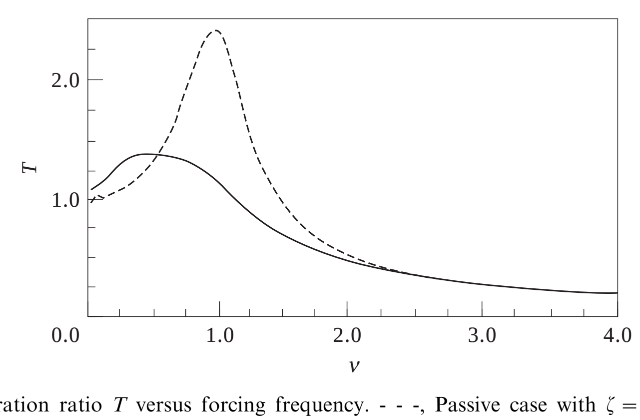

Figure 5. Acceleration ratio T versus forcing frequency. - - -, Passive case with $\zeta = 0{\cdot}25$; —, Semi-active case with $\zeta_{min} = 0{\cdot}25$, $\alpha_{dem} = 0{\cdot}45$.

and $r(t)$ is the excitation induced by the road profile for a constant vehicle speed. Using the notation of equation (2.15) and

$$\gamma = M_2/M_1, \qquad \chi = K_1/K_2, \qquad y(\tau) = x(\tau/\omega)/a, \qquad z(\tau) = r(\tau/\omega)/a, \tag{7.3}$$

equations (7.1) can be written in the dimensionless form

$$\begin{aligned} y_2'' + \delta(y_1, y') + y = 0, \\ y_1'' - \gamma\delta(y, y') - \gamma(y) + \gamma\chi(y_1 - z) = 0, \end{aligned} \tag{7.4}$$

where

$$\delta(y, y') = 2\zeta_{min}(y') + [1 - \mathrm{sgn}\,(y)(y')]\alpha_{dem}|y|. \tag{7.5}$$

Equations (7.4) and (7.5) assume instantaneous switching of the damper force from zero to its demanded value $2\alpha_{dem}|y|$ and conversely. However physically constructed, the control system will have to adjust continuously. A finite switching time is inevitable and the switching process should be included in the analysis.

The hydraulically operated system studied by the authors is shown in schematical form in Figure 7. An actuator controlled by a servo valve applies a desired normal force to the friction plates, one of which is fixed to the sprung mass and the other to the unsprung mass. The semi-active damper force can be used in an "on-off" manner ($\zeta_{min} = 0$) when the following first order equations hold:

$$\tau_c\delta' + \delta = 2\alpha_{dem}y \qquad \text{if} \qquad yy' < 0, \tag{7.6}$$

$$\tau_c\delta' + \delta = 0 \qquad \text{if} \qquad yy' > 0, \tag{7.7}$$

where $\tau_c = \omega_n T_c$. The switch response can be improved by introduction of a derivative term. The switching dynamics are now governed by following equation:

$$\tau_c\delta' + \delta = 2\alpha_{dem}[T_c y' + y]. \tag{7.8}$$

7.2. NUMERICAL RESULTS

Parameters values assumed were typical of a UK family saloon with four occupants: $M_2 = 230$ kg, $M_1 = 23$ kg, $K_2 = 23$ kN/m, $K_1 = 153$ kN/m. The natural frequency of the sprung mass in bounce is 1·59 Hz and the wheel hop frequency 13·9 Hz. The r.m.s. response of the quarter car to sinusoidal inputs was obtained as a function of frequency.

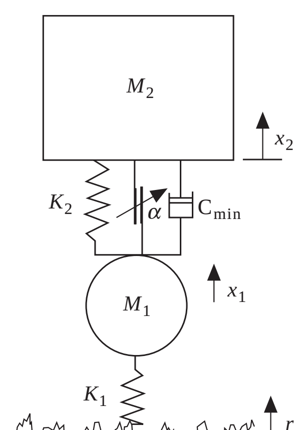

Figure 6. Quarter-car model.

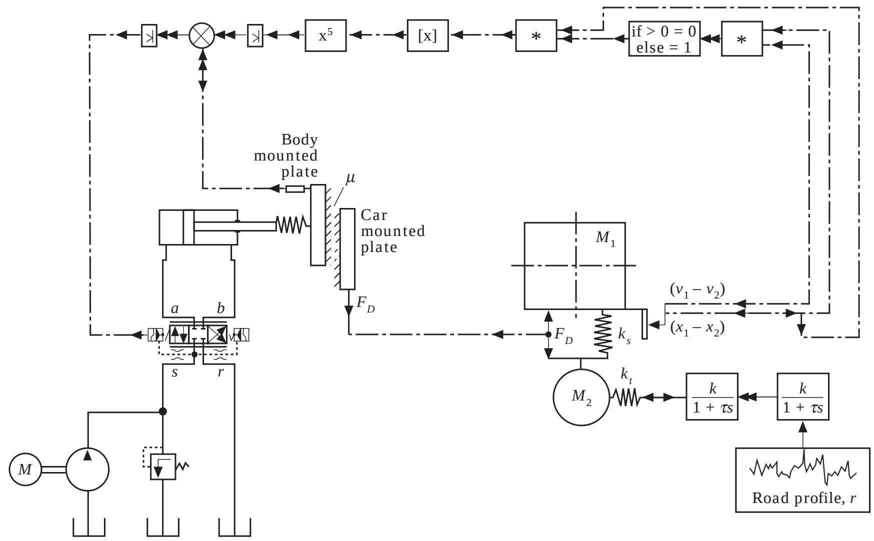

Figure 7. Semi-active friction damper and hydraulic circuit.

Road input amplitude was scaled to produce the same peak velocity at all frequencies, which is approximately the case for road profiles. The demanded α was 0·5 (i.e., full spring cancellation).

A comparison was made of the response of the system under semi-active control with the passive case, assuming a dual valued damper (low on closure, high on recoil). Figure 8a shows the comparison for r.m.s. body (sprung mass) acceleration over a period of 10 s and Figure 8b the dynamic tyre force coefficient. (The integration period was 12 s with the first two seconds of the response being ignored). Added viscous damping was considered. Note that with system considered by the authors the additional 'viscous' damping does not require another damper but can be introduced simply by control of the normal force applied to the friction plates. For Figures 8a and 8b the added viscous damping ratio in the semi-active friction case is 0·1. This low value produces vibration isolation superior to the passive case over the entire frequency range, with the exception of the secondary resonance, where the semi-active system is no worse. (The reason for the

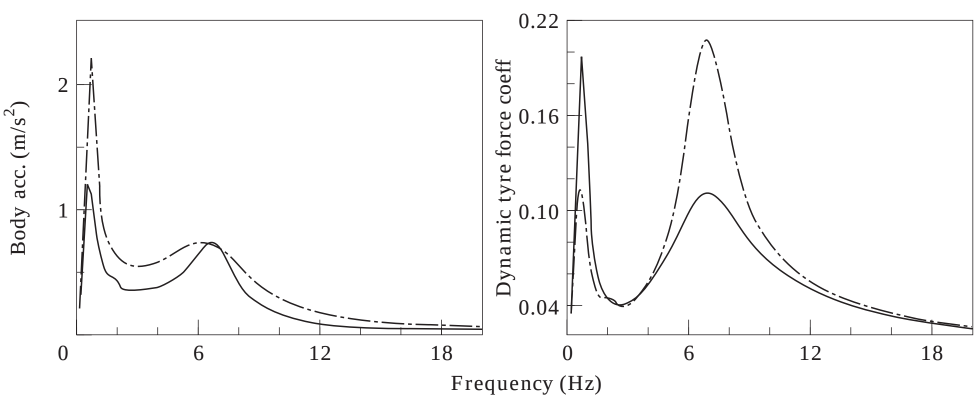

Figure 8. (a) Body r.m.s. acceleration sinusoidal input, constant peak velocity; ——, passive, $\zeta = 0{\cdot}25$; —-—, semi-active $\zeta = 0{\cdot}1$; 2DOF: (b) dynamic tyre force coefficient, sinusoidal input, constant peak velocity; ——, passive, $\zeta = 0{\cdot}25$; —-—, semi-active $\zeta = 0{\cdot}1$; 2DOF.

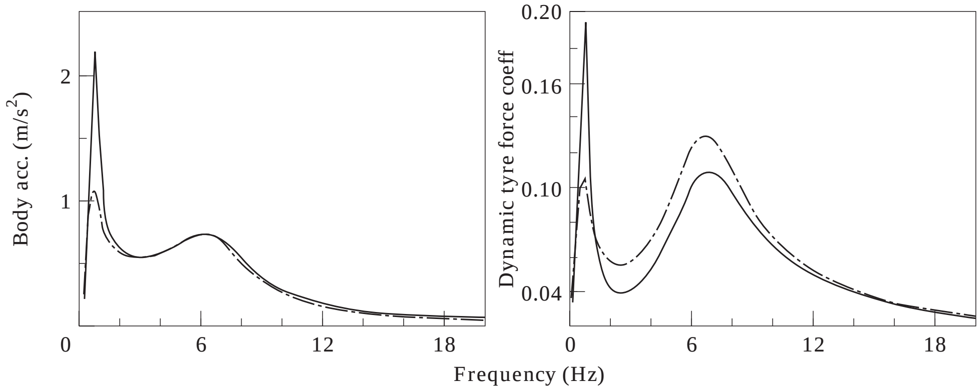

Figure 9. (a) Body acceleration, sinusoidal input, constant peak velocity; —,passive, $\zeta = 0{\cdot}25$; —-—, semi-active $\zeta = 0{\cdot}2$; 2DOF: (b) dynamic tyre force coefficient, sinusoidal input, constant peak velocity; —, passive, $\zeta = 0{\cdot}25$; —-—, semi-active $\zeta = 0{\cdot}2$; 2DOF.

peak occurring at 7 Hz rather than around 13 Hz is due to the constant peak velocity road input). However, Figure 8b shows that for this value of added viscous damping the semi-active system exhibits a large tyre force fluctuation around 7 Hz. The first peak is actually about half that in the passive case, so the overall performance of the semi-active system in regard to tyre force is not actually much worse than that in the passive case. However, a larger value of added damping appears desirable.

The corresponding plots for an added damping ratio of 0·2 are given in Figures 9a and 9b. The secondary tyre force resonance is now controlled (Figure 9b) but the acceleration of the sprung mass (Figure 9a) is somewhat increased by the doubling of the added viscous damping. Nevertheless, the performance is still distinctly superior to the passive case, as one would expect with a (semi) active system. The response of the semi-active system at the first resonant frequency is about half that of the passive system.

The behaviour of the friction damper at the resonant condition is indicated in Figure 10 for the case of $\tau_c = 0{\cdot}05$. The damper cancels the spring force over a quarter cycle, and

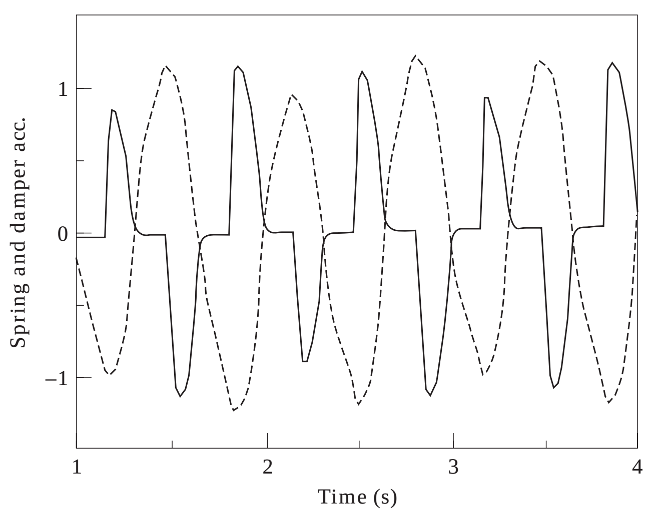

Figure 10. Harmonic road input. 1·5 Hz. Damping (——) and spring (–––) force, $\tau_c = 0{\cdot}05$, 2DOF.

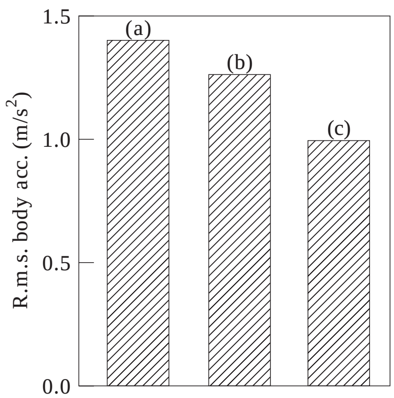

Figure 11. Random road input, 2DOF (a) passive damper $\zeta = 0{\cdot}1$, $0{\cdot}4$; (b) continuously variable viscous damper $0{\cdot}1 < \zeta < 0{\cdot}4$ (balance logic control); (c) friction damper with added viscous damping $\zeta = 0{\cdot}2$ (balance logic control); switch time constant 10 ms in each case.

is then switched off in the next quadrant. The net acceleration is halved. Finally, an integrated white noise input was used to represent the road displacement.

In Figure 11 the body acceleration achieved with a semi-active dry friction damper under full balance control ($\alpha_{dem} = 0{\cdot}5$) is compared with that produced by a conventional passive viscous damper and also with a continuously variable viscous damper. If the hydraulic system of Figure 7 is employed, the lower value of the semi-active damper force is zero, and the limit on the upper value is set by the maximum permissible spring compression. The passive damper was given a value of 0·1 on closure and 0·4 on recoil. All three systems were taken to have the same time constant (10 ms). In the passive case the switch is between the recoil and the closure settings. The body acceleration for the continuously variable damper is 12% lower than in the passive case, while for the semi-active friction damper with added viscous damping ratio of 0·2, the reduction is 29%. For a single-degree-of-freedom model with sinusoidal relative motion and instantaneous switching, a reduction of approximately 50% in body acceleration is predicted (Figures 4a and 4b as discussed

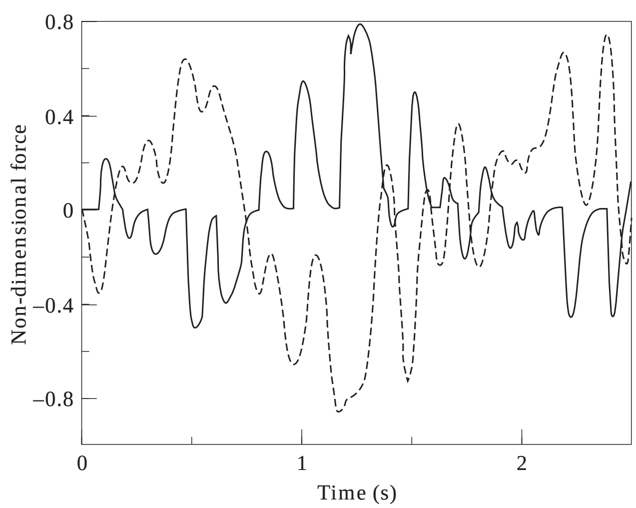

Figure 12. Generated friction force (—) versus spring force (---); time constant 10 ms, $\alpha_{dem} = 0{\cdot}5$. Zero added viscous damping, 2DOF.

in section 6.1). With a two-degree-of-freedom model, random excitation and finite switching times the reduction is around 30%.

The balance logic is evident in Figure 12 in which the spring and damper 'forces' (actually accelerations) are compared for the semi-active damper. The damper force envelope approximates to a mirror image of the spring force. However, because of higher frequency velocity components, the velocity reverses frequently, which means that the damper has to turn on and off several times between two successive zero crossing of the relative displacement. The result is a higher frequency oscillation of the actuator and spring of the friction damper. The comparison between this figure and Figure 10 for a sinusoidal input is a valuable one. The indication is that with pseudo-random inputs, to improve performance, the damper may need to be fed a filtered velocity signal.

8. CONCLUSIONS

1) As has been established by other workers, the acceleration experienced by a system controlled by a semi-active damper can be appreciably lower than that of a passive system.

2) A controlled friction device is superior to a continuously variable viscous damper since the former is able to generate a damping force when the relative velocity between body and wheel (or machine and foundation) is very low.

3) The appeal of a semi-active friction damper is that it can be utilized to achieve virtually any desired control logic by appropriate adjustment of the force on the plates. This enables hybrid logic to be employed if desired.

4) The work presented here indicates that when used with "balance" logic the addition of a modest level of simulated viscous friction, the r.m.s. sprung mass acceleration at the fundamental frequency of the passive system can be reduced by nearly 50%. For random inputs the reduction is about 30%.

5) The same logic can be applied to the reduction of the dynamic tyre force since the force fluctuation on the wheel is simply that on the sprung mass.

9. FURTHER WORK

A key factor is the performance of such a system in practice. This is the object of the current research effort. The system being built is that of Figure 7, namely a hydraulically operated actuator, controlled by a servo valve, applying a demanded force to friction plates. One concern is the variation of friction force with relative velocity and possible occurence of slip stick oscillations. Friction plate wear will also be studied. Closed and open loop control strategies will be employed.

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APPENDIX: NOTATION

C	damping constant
F_d	damping force
K	suspension stiffness
M, M_2	sprung mass
M_1	unsprung mass
T	acceleration ratio
T_c	time constant
x	relative displacement
x_1, x_0	absolute, imposed displacement
y	non-dimensional relative displacement
y_1	absolute non-dimensional displacement
Y_0	amplitude of imposed motion
Z	amplitude of excitation
α	balance parameter
χ	spring ratio
δ	total damping force
γ	mass ratio
v	forcing frequency/natural frequency
τ_c	non-dimensional constant
ζ	viscous damping ratio